On Sraffa's *q*-System[*]

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ABSTRACT:
We propose a novel interpretation of Piero Sraffa's (1960) Standard commodity: The solution price vector of his subsistence production system is invariant to an equivalence transformation of the constants of the problem only in the case of the Standard system.

JEL CLASSIFICATION CODES: ***B31, C62, E11***

KEY WORDS: ***Standard Commodity; tâtonnement***

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1. INTRODUCTION

The opening salvo in one of the most deceptively terse assaults on received economic theory in our times is fired early in the preface of Sraffa's *Production of Commodities by Means of Commodities* (1960). "No changes in output … are considered, so that no question arises as to the variation or constancy of returns" (Sraffa, 1960, p.v). Furthermore, the classical approach is contrasted with the "'marginal'" method which "requires attention to be focused on change" (Sraffa, 1960, p.v). Thereafter, Sraffa proceeds to construct input-output models in classical fashion. We present his model of "Production with Subsistence" in vector-matrix form in the next section. The literal translation is not to be found, expositors moving to the schemes of expanded reproduction of the following chapters. In a departure from Sraffa's procedure in the book, (and the general equilibrium next step anywhere), however, we do not pick a commodity as a numéraire. In addition, we traverse the second chapter with its variables and equations.

The theme of change is revisited in the opening lines of the chapter of interest, "The Standard commodity", in the form of price changes. They are price movements brought about by a change in distribution. Also, in the case of a price fluctuation, Sraffa's problem is to distinguish between a change brought about by a variation in the price of the commodity of interest and the measuring rod. Since we are concerned neither with standards of value nor distribution, our question is: How, otherwise, and if at all, might prices change? Our answer lies in the equivalence transformation of the constants of the problem. We show that the solution price vector changes pari passu only in the case of the Standard system. The theorem is proved in the following section. The relationship with the existing literature is surveyed in Section III. The implications for the critique of general equilibrium economics follow. A final section is a summary.

1. THE RESULTS

Sraffa's economy consists of commodities, '*a*', '*b*', …,'*k*', each of which is produced by a separate industry. The quantity of '*a*' produced annually is *A*; *B* the similar quantity of '*b*' and so

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[*] Ajit Sinha was most generous with his gift of the idea

on. The quantities of 'a', 'b', …, 'k' annually used by the industry that produces A is A_a, B_a, …, K_a; and A_b, B_b, …, K_b the corresponding quantities used for producing B; etcetera. These numbers are known. The unknowns to be determined are p_a, p_b, …, p_k , the prices of the commodities 'a', 'b', ...,'k' respectively.

$$\text{Let } A = \begin{Vmatrix} A_a & A_b & \cdots & A_k \\ B_a & B_b & \cdots & B_k \\ \vdots & \vdots & \vdots & \vdots \\ K_a & K_b & \cdots & K_k \end{Vmatrix} \quad p = \begin{Vmatrix} p_a \\ p_b \\ \vdots \\ p_k \end{Vmatrix} \quad Q = \begin{Vmatrix} A & 0 & \cdots & 0 \\ 0 & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K \end{Vmatrix}$$

The conditions of production appear as follows, where A^T is the transpose matrix:

$$A^T p = Qp \qquad (1)$$

Since the system is assumed to produce just enough to maintain itself, the following condition has to be met by the given matrices:

$$Ae = Qe \qquad (2)$$

where e is a unit column vector with k elements.

Sraffa disciples, studying his unpublished work recently made available, claim that the Master did not regard the solution of his "first equations" as trivial (de Vivo, 2003; Garegnani, 2005; Kurz & Salvadori, 2005). In fact, his objective was the accomplishment of the task the classical economists could not complete in the absence of the required mathematical technology. The elaboration of a theory of value on the basis of production viewed as a circular flow could now be addressed with the tool of simultaneous equations. As early as November 1927, he notes that it is necessary to be aware of the properties of the solution set. He observes that the system admits of an infinite number of solutions all of which, however, are proportional to each other. These proportions he terms absolute values.

Our theorem then is

Proposition 1. *The absolute values of (1) and (2) are not invariant to an equivalence transformation of the coefficient matrices*.

Proof. We transform the 'old' basis into a 'new' basis, the vectors and matrices in the new basis being given by starred values. In terms of the new coordinates, then, equation (1) becomes

$$A^{T*} p^* = Q^* p^* \qquad (3)$$

Now,

$$A^* = U^{-1}AU \text{ and } Q^* = U^{-1}QU \qquad (4)$$

where U is the matrix transforming the old basis vectors into the new.
Substituting this equation in (2), we get

$$UA^*U^{-1}e = UQ^*U^{-1}e \qquad (5)$$

Denote $U^{-1}e$ by W. Note, as earlier, the column k-vector W is no more than the sum of the rows of U^{-1}. Now, multiply the above equation on the right by any arbitrary non-null row k-vector S. Thereby, WS, now, is a k-dimensional matrix. Denote it by V. Matrices A^* and Q^* are now equivalent. That is to say,

$$UA^*V = UQ^*V \qquad (6)$$

However, the solution price vector associated with the transformed system (3) and (6) is indeterminate. Recall the arbitrary nature of the vector S.

The problem of constructing a Standard commodity is for us to find a set of k multipliers, q_1, q_2, …, q_k, to solve the following "q-system" (Sraffa, 1960, p.24).

$$Aq = Qq \qquad (7)$$

where the matrices A and Q are as before. This equation replaces equation (2). The corollary to our earlier result is

Proposition 2. *The solutions of the Standard system, equations (1) and (7), are invariant to an equivalence transformation of the coefficient matrices*.

Proof. The derivation of equation (3) goes through in the identical manner. In similar fashion we consider the linear operator A^* and the vector q^* in the new basis. The solution of (7) is now

$$A^*q^* = Q^*q^*$$

3. (DIS)CONNECTIONS WITH THE LITERATURE

Our reading of Sraffa diverges completely from the interpretation of his critics as well as some leading followers who propose that the Standard commodity is an expendable device.

Grounds for meeting can be found in the view that the Standard system was not constructed to validate the labour theory of value or anything in particular. Indeed, a distinction is sought between Marx's intentions in this regard and that of Sraffa (Gilibert, 2003). Marx was concerned with the physical properties among industries which were essential for the smooth development of capitalist production. Sraffa, on the other hand, saw in Marx's schemes of reproduction important repercussions for the process of price determination. Continuing along this trajectory of sympathy, we end with the congruence of our result with the comment of Sraffa's literary executor that the Standard commodity is one expression of a system of coordinates in a space and serves the abstract task of clarifying the properties when changes in the coordinates are possible (Garegnani, 2000, p.58). Another account which maps, at most points, onto our discussion, except for the view that the Standard commodity is not a construct to be used for investigation, is the essay by Nisticò and Rodano (2005). According to them, the theory of prices is the "'hard core'" of Sraffa's analysis (Nisticò and Rodano, 2005, p.474). In a model with given quantities, prices must be the sole object of analysis. The prices of the first chapter of Sraffa's book are "'necessary real costs'" which are unavoidable if the economy is to be reproduced (Nisticò and Rodano, p.475). In an endorsement of the section to follow, they suggest that Sraffa's treatise strikes at the soft core of one of the foundations of the general equilibrium model, the (in)stability of the process of groping towards equilibrium.

4. "…A CRITIQUE OF ECONOMIC THEORY"

Defenders of the neoclassical canon like Hahn and Samuelson frequently protest that the economic theory against which Sraffa's contribution is a counterpoise is a straw man. A modern user of general equilibrium theory, goes the oft-repeated complaint, would find the terms and conditions of Sraffa's program incomprehensible. The paper suggests, on the other hand, that Sraffa's riposte can be given a contemporaneity and forward look that it allegedly lacks. Specifically, the charge against the general equilibrium model that endures is the instability of the tâtonnement process. Up until the present, the device of the auctioneer has been indispensable in the dynamics of the familiar model of decentralized exchange in which prices and quantities move towards market-clearing levels.

5. CONCLUSION

We proffer a 'minimally indecomposable' interpretation of Sraffa's classic. It turns out that the returns, all the same, are strictly increasing. The solution price vector in the case of a classical stationary system changes arbitrarily in response to an equivalence transformation of the givens of the problem. Only Sraffa's Standard system can deliver the required invariance. Since the 'higgling of the market' in neoclassical economics is, in general, unstable, absent the introduction of strong and unrealistic assumptions, the "critique of economic theory" can move, along these lines, beyond a "prelude". The future might lie in the extension of the device of the Standard system to capture the insights of Sraffa's book following the first chapter.

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